Math 347: Lecture 4 - Worksheet
September 10, 2018

1) Consider $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ the set of natural numbers, $S = P(\mathbb{N})$ the power set of $\mathbb{N}$, $f : \mathbb{N} \to \mathbb{N}$ a function given by $f(n) = n^2 + 1$. Are the following true or false? Justify.

    (i) $1 \subset P(\mathbb{N})$?

    (ii) $1 \in \mathbb{N}$?

    (iii) $1, 2 \in P(\mathbb{N})$?

    (iv) $2 \subset P(\mathbb{N})$?

    (v) $f^{-1}(0) \in \mathbb{N}$?

    (vi) $f^{-1}(1) \in \mathbb{N}$?

    (vii) $f^{-1}(f(1, 2)) \subset \mathbb{N}$?

    (viii) $1, 2 \subset \mathbb{N}$?

    (ix) $\mathbb{N} \subset P(\mathbb{N})$?

    (x) $\emptyset \in P(\mathbb{N})$?

2) Consider

$$P(x) = \text{"}x^2 \text{ is positive."}, \quad Q(x) = x^2, \quad R = \text{"all real numbers are positive."}$$

Are the following true or false? Justify.

    (i) $P(x)$ is a statement.

    (ii) $(\forall x \in \mathbb{R})P(x)$ is a statement.

    (iii) $(\exists x \in \mathbb{R})P(x)$ is a statement.

    (iv) $Q(2)$ is a statement.

    (v) $Q(2) = 5$ is a statement.

    (vi) R is a statement.

    (vii) $R \Rightarrow (Q(2) = 5)$ is true.

    (viii) $(x \in \mathbb{R}) \Rightarrow P(x)$.

3) What is the contrapositive of the statement: "For $f(x) = x^2 + b$ if $x \neq y$ and $x \neq -y$, then $f(x) \neq f(y)$."? Prove that the contrapositive is true. What can you deduce?

4) Prove the statement "If a is bigger than or equal to any real number smaller than b, then $a \geq b$." Consider proof by contradiction.

5) Consider the numbers $x_1, \ldots, x_4$. Prove that at least one of them is smaller than or equal to their average.

6) (Challenge) Suppose that a and b are integer numbers not divisible by 3. Prove that $ax^2 + b = 0$ has a rational solution if and only if a and b have the same reminder when divided by 3, i.e. $a \equiv b \pmod 3$.